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Monte Carlo Based Sampling Distribution of Annual Rate of Exceedance for Earthquake Insurance

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ABSTRACT

Seismic hazard expressed in terms of annual rate of exceedance and is used to calculate the earthquake insurance premium. Annual rate of exceedance is a complicated function of magnitudes, distances from site to earthquake sources and attenuation. Due to its complexity, determination of exact sampling distribution of earthquake insurance premium is not an easy task. This research proposes Monte Carlo simulation approach to determine the sampling distribution of earthquake insurance premium. Annual rate of exceedance was simulated first and then the insurance premium calculated based on simulation of annual rate of exceedance. The simulation involves quantifying synthetic catalogue similar to historical catalogue. Its simulation is conducted in order to construct annual rate of exceedance as an indicator of earthquake risk used in earthquake insurance. The simulation of 25 iteration and sample size of 100 shows that the sampling distribution of insurance premium is skewed to the right. The idea of using Monte Carlo simulation to study the sampling distribution of annual rate of exceedance is the originality of the study and the study contributes to the methodology of earthquake insurance.

Keywords: Annual Rate Of Exceedance, Monte Carlo Simulation, Sampling Distribution, Synthetic Catalogue

1. INTRODUCTION

Seismic hazard is quantified by the probability that the ground motion at a site will exceed a value at least once in a given time period. Probabilistic seismic hazard analysis (PSHA) is the probabilistic method based on the law of total probability (LTP). The probability of a peak ground acceleration (PGA) value at site can be calculated by multiplying the conditional probability of obtaining that PGA from an earthquake of given magnitude at a given distance and integrating over all possible values of distance and magnitude (Musson, 2000). The seismicity is characterized by annual rate of exceedance.

Monte Carlo simulation has been used for simulated complicated process: determine the materials for neutron shielding (Paul et al., 2023), fatigue crack growth (Giannella, 2022), and risk of construction project (Hendradewa, 2019). The methods are a class of computational algorithms that rely on repeated random sampling. There are two main situations where Monte Carlo simulation is used: complicated model to determine the quantity, and it is not possible to determine analytically the quantity of interest. Monte Carlo computation of annual rate of exceedance consist of selecting an epicenter from an observed earthquake catalogue, then randomly select a magnitude from a magnitude list. The distance from the selected epicenter to the site is calculated and the used to find ground motion at the site. The set of ground motion above the threshold are used to compute hazard values at several different level of ground motion. One hundred synthetic catalogue of 273 events were generated using Monte Carlo simulation for seismic hazard analysis (Ebel & Kafka, 1999).

Many experts have advocated the use of Monte Carlo simulation in quantifying the seismic hazard distribution to be expected for a region. Osei et al., (2018) applied the Monte Carlo approach in quantifying the seismic hazard. Alessandri et al., (2018) proposed a new tool for the probabilistic seismic hazard assessment based on Monte Carlo simulations. Monte Carlo simulation was used to predict the probability that the nuclear power plant reaches an unsafe state (Ferrario, 2014). Akkar and Cheng (2016) shows the implementation of Monte Carlo simulation for computing the annual exceedance rates of dynamic ground motion intensity e.g. peak ground acceleration, PGA. Kowsari and Halldorsson (2017) used Monte Carlo simulation to generate the earthquake catalogue. Monte Carlo simulation have been used to produce synthetic catalogue for probabilistic seismic hazard assessment. The study produces the seismic hazard zoning, hazard curves and deaggregation of seismic hazard (Amerian et al., 2019). Monte Carlo methodology have been developed for the estimation of seismic hazard at a site using a resampling of an earthquake catalogue to construct synthetic earthquake catalogue and find earthquake ground motion from the hazard values (Ebel & Kafka, 1999).

Earthquake insurance premium, EADR, is computed using annual rate of exceedance, and it is not possible to determine the sampling distribution of this quantity. This study proposes a Monte Carlo simulation approach for the determination of sampling distribution of annual rate of exceedance and expected annual damage ratio. The input for the simulation consist of study area contains of point location i.e. longitude, and latitude, the earthquake parameters comprised of the earthquake source, the fault type, $m_{\min}, m_{\max}$, values of Gutenberg-Richter parameters, magnitude distribution, and the attenuation distribution. The output file contains the annual rate of exceedance that individual target accelerations will be exceeded. At any site, this data is useful to illustrate the seismic hazard curve which represents the probability of exceeding different PGA at a given site. Based on Monte Carlo simulation of annual rate of exceedance, the sampling distribution of earthquake insurance premium can be determined empirically.

2. RESEARCH METHOD

The research methodology used in this study consists of the following steps: 1. Basic theory of magnitude distribution, annual rate of exceedance and expected annual damage ratio 2. Describes the area of study, define the earthquake catalogue 3. Find the earthquake seismic parameter to generate the synthetic earthquake catalogue using Monte Carlo simulation 4. Calculate the annual rate of exceedance for each simulated synthetic earthquake catalogue 5. Based on the annual rate of exceedance calculated at step 3, calculate the expected annual damage ratio, EADR 6. Plot the annual rate of exceedance and the expected annual damage ratio. Boxplot and histogram of EADR represent the empirical sampling distribution of EADR.

The probability density function of earthquake magnitude follows the Gutenberg-Richter recurrence law $N(\geq m) = 10^{a-bm} = e^{\alpha-\beta m}$ where $N(\geq m)$ is the number of earthquakes that are larger or equal to a given magnitude m, and α and β are the Gutenberg-Richter parameters, $\alpha = 2.303a$ and $\beta = 2.303b$. The earthquake premium is correlated with the pay-off of the insured financial assets, and should be calculated based on the frequency and the severity of earthquakes (Deniz & Yücemen, 2007). The earthquake damage is correlated with the modified Mercalli intensity (MMI). The seismic hazard scale computed in terms of peak ground acceleration (PGA) are converted to MMI scale using empirical relationship (Trifunac & Brady, 1975). $EADR$ corresponds to the insurance rate for a unit property replacement cost, and is used as a measure of magnitude of earthquake damage and defined as equation (1).

$$EADR = \sum_{MMI} MDR(MMI) \times SH(MMI) \quad \text{... (1)}$$

where MDR denotes the mean damage ratio and SH denotes the annual probability of an earthquake of intensity MMI occurring at the assets. The seismic hazard, SH, is related to annual rate of exceedance $\lambda(PGA > x)$ times per year. Since the insurance rate is related to the annual rate of exceedance, the focus of the study is to develop Monte Carlo simulation for annual rate of exceedance.

The annual rate of exceedance of $PGA > x$ considers all sources is the sum of the rates for each individual as shown in equation (2)(Baker, 2008).

$$\begin{aligned}\lambda(PGA > x) &= \sum_{i=1}^{n_{sources}} \lambda\left(M_i > m_{\min}\right)\int_{m_{\min}}^{m_{\max}}\int_{0}^{r_{\max}} P\left(PGA > x \mid m, r\right) f_{M_i}\left(m\right) f_{R_i}\left(r\right) dr dm \\ &\approx \sum_{i=1}^{n_{sources}} \lambda\left(M_i > m_{\min}\right)\sum_{j=1}^{n_M}\sum_{k=1}^{n_R} P\left(PGA > x \mid m_j, r_k\right) P\left(M_i = m_j\right) P\left(R_i = r_k\right)\end{aligned} \quad \text{... (2)}$$

where $n_{sources}$ is the number of sources considered, M_i and R_i denote the magnitude and the distance for source i. The range possible M_i and R_i have been discretized into n_M and n_R intervals. It integrates the knowledge about rates of occurrence, the possible magnitudes, and the distribution of ground shaking due to earthquakes. Predictive model for the mean in peak ground acceleration was proposed by Cornell at all. (1979) in Baker (2008), with mean and standard deviation $\mu_{\ln PGA} = -.152 + .859M - 1.803\ln(R + 25), \sigma = .57$. The logarithm of PGA, $\ln PGA$, follows normal distribution, the probability of exceeding any PGA can be calculated using cumulative normal distribution Φ shown in equation (3) given magnitude m and distance from site to the source r.

$$P\left(PGA > x \mid m, r\right) = 1 - \Phi\left(\frac{\ln x - \mu_{\ln PGA}}{\sigma_{\ln PGA}}\right) \quad \text{... (3)}$$

The probability of exceeding 1 g, given magnitude $m = 6.5$ and distance $r = 3$ is

$$P\left(PGA > 1 \mid 6.5, 3\right) = 1 - \Phi\left(\frac{\ln 1 - (-.5765)}{.57}\right) = 1 - \Phi\left(1.01\right) = .16 \quad \text{... (4)}$$

Annual rate of exceedance, $\lambda\left(PGA > x\right)$, equation (2), is a complicated model. It involves the magnitude distribution, the distance distribution from the site to the earthquake source, ground motion prediction equation and earthquake sources. It is not possible to determine analytically the sampling distribution of annual rate of exceedance. The earthquake insurance premium is a function of annual rate of exceedance, it is a complicated task to determine premium rate. Monte Carlo simulation is proposed in this study to find the sampling distribution of earthquake insurance premium.

3. RESULT AND DISCUSSION

Annual rate of exceedance summation of equation (2) can quickly get lengthy, and is preferred using computer software. Examples of using equation (2) for simple seismic case study will be discussed in the next section, see (Baker, 2008) for details. Consider an earthquake sources consist of two fault: fault A and fault B. Fault A produces earthquake with magnitude of scale 6.5 and site to site distance of 10 km. Assume that earthquakes occur at a rate of $\lambda = .01$ times per year, the return period of this fault is 100 year. Fault B produces earthquakes of magnitude 7.5 at a distance of 20 km at rate of $\lambda = .002$ times per year. Rate of exceedance for fault A, as function of *PGA* is

$$\begin{aligned}\lambda\left(PGA > x\right) &= \lambda\left(M > m_{\min}\right) P\left(PGA > x \mid 6.5, 10\right) P\left(M = 6.5\right) P\left(R = 10\right) \\ &= .01 P\left(PGA > x \mid 6.5, 10\right) \\ &= .10\left[1 - \Phi\left(\frac{\ln x - \mu_{\ln PGA}}{\sigma_{\ln PGA}}\right)\right] = .10\left[1 - \Phi\left(\frac{\ln x - \ln .3758}{.57}\right)\right]\end{aligned}$$

The annual rate of exceeding 1 g is

$$\lambda\left(PGA > 1g\right) = .01 P\left(PGA > 1g \mid 6.5, 10\right) = .01\left[1 - \Phi\left(1.72\right)\right] = .01 \times .044 = .0044$$

Given an earthquake on fault B, the probability of $PGA > .3758g$, is

$$P(PGA > .3758g \mid 7.5, 20) = 1 - \Phi\left(\frac{\ln .3758g - \ln .5639}{.57}\right) = .761$$

Summation over all sources, the overall rate of $PGA > .3758g$ is

$$\lambda(PGA > .3758g) = .01 \times P(PGA > .3758g \mid 6.5, 10) + .002 \times P(PGA > .3758g \mid 7.5, 20)$$
$$= .01 \times .5 + .002 \times .761 = .00652$$

The two computed annual rates are plotted in Figure 1. The figure shows the annual rates for each fault and the overall fault. Fault A contributes much more to the overall rate of exceedance due to higher rate of fault A.

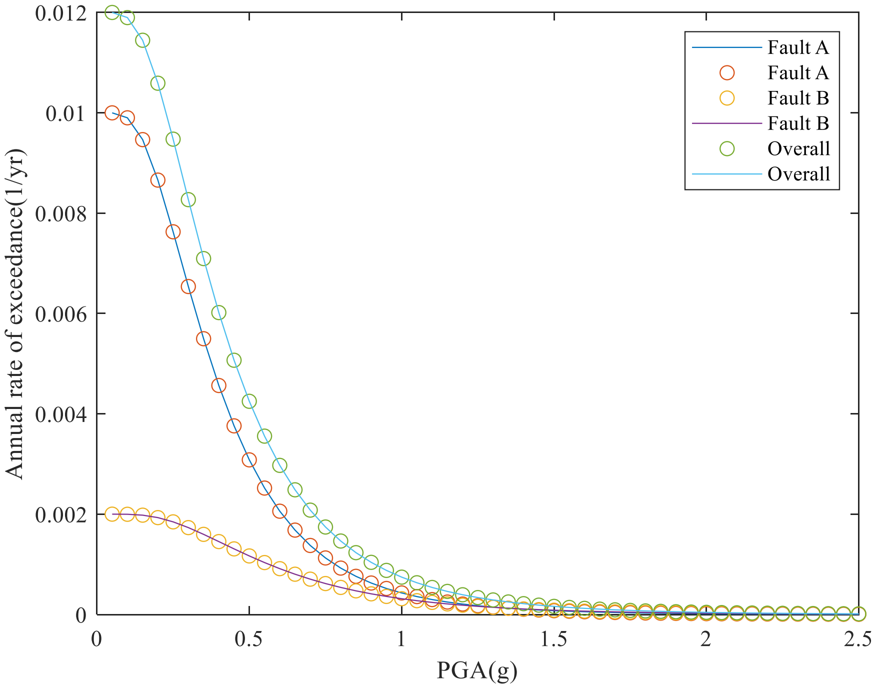

Figure 1. Annual rate of exceedance for the two sources for a site. Fault A contributes much more to the overall rate of exceedance due to higher rate of fault A

The previous section was a study of annual rate of exceedance for simple case; one site and two faults. The next section will discuss annual rate of exceedance for a study area with more than two sites. The study area and data sources used in this study is shown in Figure 2 and was taken from ISC website. The study area is located in west Java. Figure 2 shows the location of earthquake epicenter of 53 earthquakes event with minimum magnitude of 4.3 and maximum of 6.3. A matlab script CU-PSHA (Pailoplee & Palasri, 2014) was used for the computation of annual rate of exceedance. Figure 3 shows Monte Carlo simulation of annual rate of exceedance of 25 replications of sample size 100. A sample of size 100 was generated from magnitude and distance distribution produces one annual rate of exceedance. The annual rate of exceedance simulation was repeated for 25 times, and produces 25 curves of annual rate of exceedance. The annual rates of exceedance are converted to probability of exceedance $p(t) = 1 - \exp(-\lambda t)$.

Based on these 25 annual rates of exceedance, twenty five $EADR$ was calculated and boxplot and histogram of $EADR$ are shown in Figure 4. The plots show that $EADR$ distribution was skewed to the right. The range of $EADR$ values can be used as an information of range of premium rate. Figure 4 shows the sampling distribution of $EADR$ using Monte Carlo simulation. The earthquake insurance premium $EADR$ is a complicated model and it is not possible to determine the sampling distribution of $EADR$. This study proposes a Monte Carlo simulation to obtain the sampling distribution of *EADR* . The Monte Carlo simulation approach for determine the sampling distribution of annual rate of exceedance, probability of exceedance and *EADR* can be considered as a novelty in this study. The study should be extended to another area of study and the range of insurance premium, based on annual rate of exceedance, should be compared with another methods.

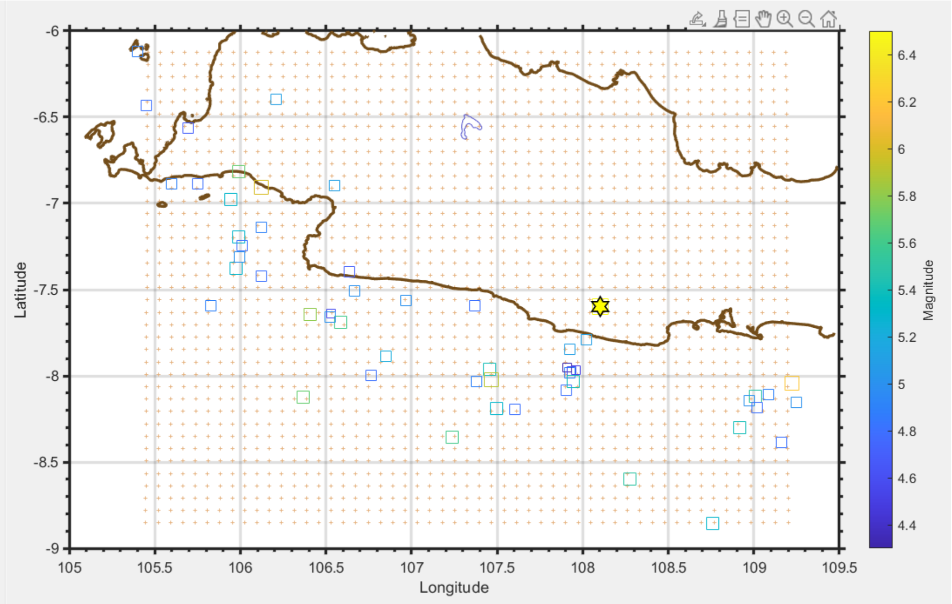

Figure 2. Earthquake locations used in the study. The study area is located in west java. The map shows all 53 earthquakes of magnitude of 4.3 to 6.3. The star shows the epicenter with maximum magnitude.

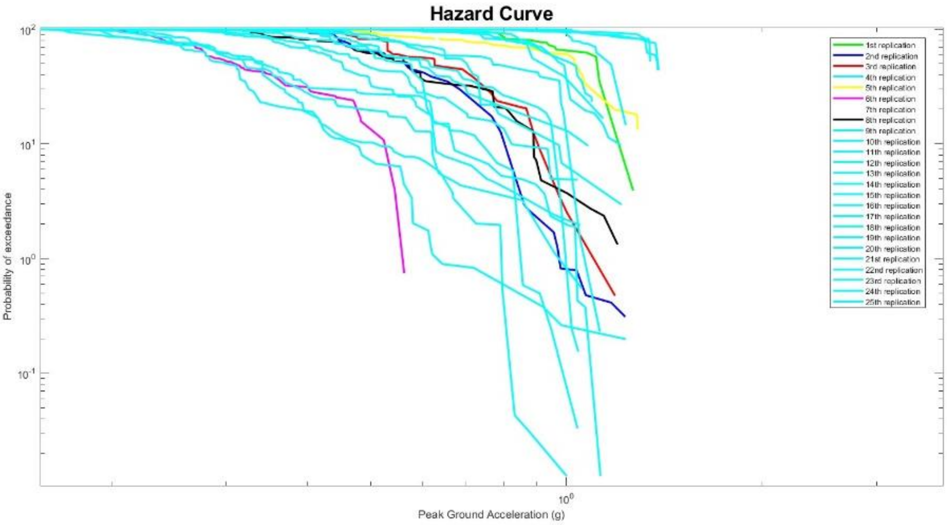

Figure 3. Monte Carlo Simulation of Probability of Exceedance for The Study Area, 25 Replications, Sample Size 100.

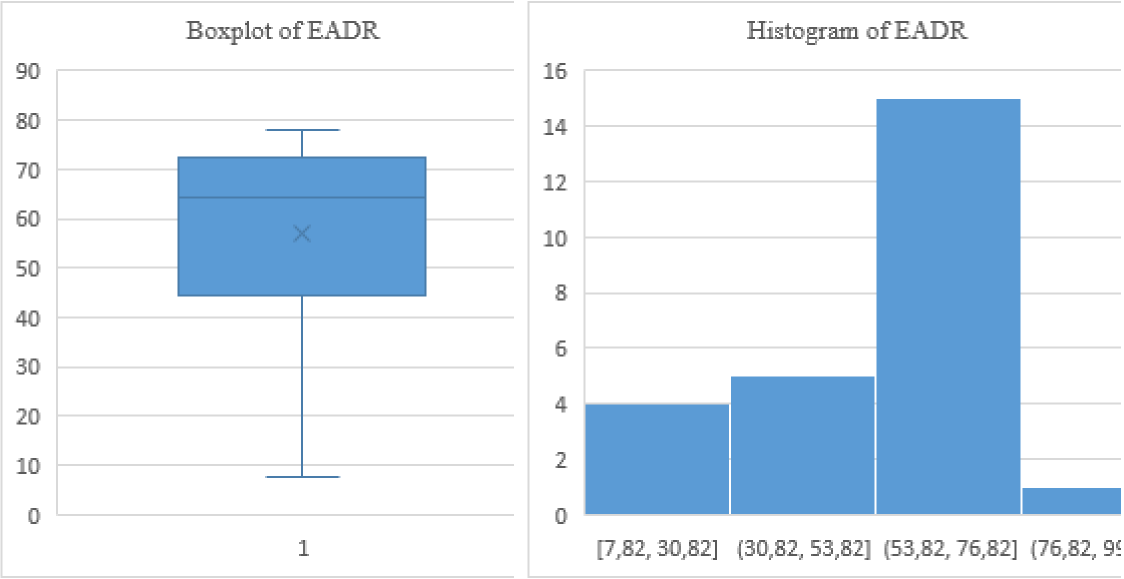

Figure 4. Boxplot and histogram of Monte Carlo simulation of earthquake insurance EADR. The plots show that the distribution is skewed to the right. The distribution informs the range of insurance premium rate for the site of interest

4. CONCLUSIONS

The determination of earthquake insurance premium is a complicated task and it is not possible to determine analytically the sampling distribution of the premium rate. In this study, Monte Carlo simulation was proposed to study the sampling distribution of annual rate of exceedance and earthquake insurance premium. The premium consists of two information: the seismic hazard of the site and the damage probability matrix due to earthquakes. The main focus of this article is on the seismic hazard analysis, known as probabilistic seismic hazard assessment (PSHA) or annual rate of exceedance. A matlab script CU-PSHA was used as a computational tool for annual rate of exceedance simulation. The results show that the Monte Carlo seismic hazard simulation can be used to explore the empirical distribution of earthquake premium rate. In this article, it is assumed that the earthquake occurrences follows homogeneous Poisson process, the methodology can extend in assuming that the earthquake occurrences follows a nonhomogeneous spatiotemporal process.

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